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# δmk-Closed Set and δmk-Open Set in Topological Spaces

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**ABSTRACT:** In this paper, we introduce and investigate the notation of  $\delta$ mk-closed set and  $\delta$ mk-open set. We obtain fundamental properties of  $\delta$ mk-closed set,  $\delta$ mk-open set and discuss the relation between  $\delta$ mk-closed set,  $\delta$ mk-open set and other related sets.

KEYWORDS: omk-closed set,omk-open set.

# **I.INTRODUCTION**

The concept of generalized closed sets introduced by Levine [1] plays a significant role in general topology. After the introduction of generalized closedsets many research papers were published which deal with different types of generalized closed sets. Y. Gnanambal [2] introduced the concept of gpr-closed set and investigated its basic properties. H. Maki et al. [3] defined the concept of gp-closed set in topological spaces and established results related to it. These concepts motivated us to define a new class of sets called the delta generalized pre-closed sets. Throughout this paper,  $(X, \tau)$  (or simply X) represents topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space  $(X, \tau)$ , we denote the closure of A, the interior of A and complement of A as Cl(A),int(A) and  $A^c$  respectively. The modified forms of generalized closed sets and  $\hat{g}$ -closed sets were studied by

Cl(A),int(A) and  $A^c$  respectively. The modified forms of generalized closed sets and  $\hat{g}$ -closed sets were studied by K.Balachandran, P. Sundaram and H. Maki [4]. In 2008, S. Jafari, T. Noiri, N. Rajesh and M.L. Thivagar [5] introduced the concept of  $\hat{g}$ -closed sets and their properties. In this paper, we introduce new classes of sets called  $\hat{g}$ -closed sets for topological spaces.

### **II.PRELIMINARIES**

**2.1**. A subset A of a topological space X is called,

pre-open [6] if  $A \subseteq int(cl(A))$  and Pre-closed if  $cl(int(A)) \subseteq A cl(int(A))$ .

**2.2**. A subset A of a topological space X is called,

 $\alpha$  - open [7] if  $A \subseteq int(cl(int(A[)))$  and  $\alpha$  - closed if  $cl(int(cl(A))) \subseteq A$ .

2.3. A subset A of a topological space X is called,

b - open [8] if  $A \subseteq cl(int(A)) \cup int(cl(A))$  and b - closed if  $cl(int(A)) \cap int(cl(A)) \subseteq A$ .

**2.4**. A subset A of a topological space X is called,

regular – open [9] if A = cl(int(A)) and regular – closed if A = cl(int(A)).

2.5. A subset A of a topological space X is called,

semi-open [10] if  $A \subseteq cl(int(A))$  and semi-closed if  $int(cl(A)) \subseteq A$ .



(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 1, January 2018

2.6. A subset A of a topological space X is called,

Semi preopen [11] if  $A \subseteq cl(int(cl(A )))$  and semi preclosed if  $int(cl(int(A ))) \subseteq A$ .

2.7. A subset A of a topological space X is called,

 $\delta$  - closed [12] if  $A = cl_{\delta}(A)$  where  $cl_{\delta}(A) = \{x \in X : int(cl(U))) \cap A \neq \Phi, U \in \tau \text{ and } x \in U \}$ . The Complement of  $\delta$ -closed is  $\delta$ -open.

**2.8**. Let  $(x, \tau)$  be a topological space and  $A \subseteq X$ .

The pre-interior of A, denoted by pint(A), is the union of all preopen subsets of A.

The pre-closure of A, denoted by Pcl(A), is the intersection of all preclosed sets containing A.

2.9. A subset A of a topological space X is called,

Generalized closed (briefly,g-closed) [1] if  $cl(A) \subseteq G$  whenever  $A \subseteq G$  and G is open in X. The complement of g-closed is g-open.

2.10. A subset A of a topological space X is called ,

Generalized  $\alpha - closed$  (briefly,  $g\alpha - closed$ )[113] if  $\alpha cl(A) \subseteq G$  whenever  $A \subseteq G$  and G is  $\alpha - open$  in X. The complement of  $g\alpha$ -closed is  $g\alpha$ -open.

**2.11**. A subset A of a topological space X is called,

 $\alpha$  - generalized closed (briefly,  $\alpha g$  - closed) [14] if  $\alpha cl(A) \subseteq G$  whenever  $A \subseteq G$  and G is Open in X. The complement of ag-closed is ag-open.

**2.12**. A subset A of a topological space X is called ,

Generalized pre-closed (*briefly*, gp - closed)[15] if  $pcl(A) \subseteq G$  whenever  $A \subseteq G$  and G is Open in X. The complement of gp-closed is gp-open.

**2.13**. A subset A of a topological space X is called,

Generalized preregular closed (*brieflygpr-closed*)[2] if  $pcl(A) \subseteq G$  whenever  $A \subseteq G$  and G is regular Open in X. The complement of gpr-closed is gpr-open.

2.14. A subset A of a topological space X is called,

 $\delta$  – generalized closed ( *briefly*,  $\delta g$  – *closed* )[16] if  $\delta cl(A) \subseteq G$  whenever  $A \subseteq G$  and G is

Open in X. The complement of  $\delta g$ -closed is  $\delta g$ -open.

**2.15**. A subset A of a topological space X is called ,

Pregeneralized closed (*briefly*, pg - closed)[3] if  $pcl(A) \subseteq G$ 

whenever  $A \subseteq G$  and G is preopen in X. The complement of pg-closed is pg-open

2.16. A subset A of a topological space X is called,

Generalized semi-preclosed (*briefly*, gsp - *closed*) [17] if  $spcl(A) \subseteq G$  whenever  $A \subseteq G$  and G is Open in X. The complement of gsp-closed is gsp-open.

**2.17**. A subset A of a topological space X is called,

Pre semi closed [18] if  $spcl(A) \subseteq U$  whenever  $A \subseteq G$  and G is

g - open in X. The complement of Pre semi closed is Pre semi open.

**2.18.** A subset A of a topological space X is called a delta generalized pre-closed (briefly,  $\delta$ gp-closed) [19] set if  $pcl(A) \subseteq G$  whenever  $A \subseteq G$  and G is  $\delta$ -open in X. The complement of  $\delta$ gp-closed is  $\delta$ gb-open.

The family of all  $\delta gp$ -closed sets in a topological space X is denoted by  $\delta GPC(X)$ .

# Example:2.1

Let  $X = \{a, b, c\}, \tau = \{\phi, x, \{a\}, \{b\}, \{a, b\}\}$   $\tau^c = \{\phi, x, \{b\}, \{ac\}, \{bc\}\}$  then  $\delta GPC(X) = \{\phi, x, \{c\}, \{a, c\}, \{b, c\}\}.$ 



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# Vol. 7, Issue 1, January 2018

**2.19.** Let  $(X, \tau)$  be a topological space. A subset  $A \subseteq X$  is said to be  $\hat{g}$ -closed set[20] if  $cl(A) \subseteq G$  whenever  $A \subseteq G$  and G is semi o-pen in X. The complement of  $\hat{g}$ -closed is  $\hat{g}$ -open. **Example:2.2** 

Let  $X = \{a, b, c\}, \tau = \{\phi, x, \{a\}, \{a, c\}\}$  and  $\tau^c = \{\phi, x, \{b\}, \{b, c\}\}$  then  $\hat{g} = \{\phi, x, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ .

#### III.AMK-CLOSED SET

### **Definition:3.1**

.Let  $(X, \tau)$  be a topological space. A subset  $A \subseteq X$  is said to be  $\delta mk - closed$  set if

(i)  $\delta gp$ -closed,

(ii)  $\hat{g}$ -closed.

The complement of  $\delta mk - closed$  is  $\delta mk$ -open.

### Example:3.1

Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, x, \{b\}, \{c\}, \{bc\}\}\$  and  $\tau^c = \{\phi, x, \{a\}, \{ac\}, \{ab\}\}\$ then  $\delta mk - closed$ 

 $= \{ \phi, x, \{a\}, \{ab\}, \{ac\} \}.$ 

# Theorem:3.1

Let  $(x,\tau)$  be a Topological Space. Prove that Every closed set is  $\delta mk - closed$  set in X.

But not conversely. (U=X)

### **Proof:**

Let A be a Closed set.

then Cl(A)=A,

Take  $A \subseteq \cup$  and U is semiopen in X.

Then  $Cl(A) \subseteq U$ 

 $\therefore$  A is  $\hat{g}$ -closed. Every Semi open is  $\delta$ -open in X. Therefore U is  $\delta$ -open in X.

 $Cl(A) \subseteq Pcl(A)$  and A is  $\hat{g}$ -closed.

Therefore A is Preclosed in X.

Therefore A is  $\delta mk - closed$  set.

# Example:3.2

Let X={a,b,c},  $\tau$ ={ $\phi$ ,x,{b},{bc}},  $\tau^{c}$ ={ $\phi$ ,x,{a},{ac}.

 $\delta$ mk-closed set ={ $\phi,x,{a},{c},{a,c}$ }, Preclosed={ $\phi,x,{a},{c},{a,c}$ },

 $\hat{g}$ -closed={ $\phi,x,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}$ }. The elements {a},{a,c} are in closed, Preclosed,

 $\hat{g}$ -closed and  $\delta$ mk-closed.

Therefore closed set is 8mk-closed set.

The converse of the above theorem is not true.



(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 1, January 2018

# Example:3.3

Let  $X = \{a, b, c\}, \tau = \{\{c\}, \{a, b\}, \phi, X\}$  and  $\tau^c = \{\{c\}, \{a, b\}, \phi, X\}$ .

 $\delta$ mk-closed set={ $\phi,x, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}$ }

closed set={ $\phi,x, \{c\}, \{a,c\}$ }. The element {a} is in  $\delta$ mk-closed but not in closed set

# Therorem3.2.

Let  $(x,\tau)$  be a Topological Space. Prove that Every open set is  $\delta mk$ -open set in X.(U=X). But not conversely.

# Exapmle:3.4

Let X={a,b,c},  $\tau$ ={ $\phi$ ,x,{b},{bc}},  $\tau^{c}$ ={ $\phi$ ,x,{a},{ac}}.

 $\delta$ mk-open set={ $\phi,x, \{b,c\}, \{a,b\}, \{b\}$ }

The element {a,b}€δmk-open set but not in open set.

# Theroem:3.3

Let  $(x,\tau)$  be a Topological Space. Prove that Regular Closed Set is  $\delta mk - closed$  set in X. But not conversely. (U=X)

# **Proof:**

Let A be a Regular Closed set in X. Then A=Cl(Int(A)). And also A is Regular open in X. Therefore Int(A)=A  $\Rightarrow$  A=Cl(Int(A))=Cl(A)  $\Rightarrow$  A=Cl(A) Take  $A \subseteq \cup$  and U is semiopen in X. Then Cl(A)  $\subseteq U$   $\therefore$  A is  $\hat{g}$ -closed. Every Semi open is  $\delta$ -open in X. Therefore U is  $\delta$ -open in X. Cl(A)  $\subseteq$  Pcl(A)  $\subseteq$  A  $\subseteq$  U A is  $\hat{g}$ -closed. Therefore A is  $\delta$ -generalized preclosed in X. Therefore A is  $\delta mk - closed$  set in X.

### Example:3.5

Let X={a,b,c},  $\tau = \{\phi, x, \{c\}, \{b, c\}, \{a, c\}\}$  and  $\tau^c = \{\phi, x, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}, \{a, b\}, \{a, b\},$ 

 $Regular closed = \{\phi, x, \{a, c\}, \{b\}\}, Regular open = \{\phi, x, \{a, c\}, \{b\}\} and \delta mk-closed set = \{\phi, x, \{a, c\}, \{a, b\}, \{a\} \{b\}\}.$ 

Here Regular closed set = Regular open set

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Vol. 7, Issue 1, January 2018

The converse of the above theorem need not to be true.

In the above example, element {a} in  $\delta$ mk-closed set but not in Regular Closed Set.

### Theorem 3.4

Let  $(x,\tau)$  be a Topological Space. Prove that Regular Open Set is  $\delta mk$ -open set in X.(U=X). But not conversely.

### Example:3.6

Let X={a,b,c},  $\tau = \{\phi, x, \{c\}, \{b\}, \{b,c\}, \{a,b\}\}$  and  $\tau^c = \{\phi, x, \{a,c\}, \{a,b\}, \{a\}, \{c\}\}, \{a,b\}, \{a,b$ 

Regular open={ $\phi, x, \{a, b\}, \{c\}$ } and  $\delta mk$ -Open set= { $\phi, x, \{b, c\}, \{a, b\}, \{c\} \{b\}$ }.

The element  $\{b\}$  is in  $\delta mk$ -Open set but not in Regular open.

### Theroem:3.5

Let  $(x,\tau)$  be a Topological Space. Prove that  $\delta mk$ -closed Set is Semi closed set in X.

But not conversely. (U=X)

### **Proof:**

Let A be a  $\delta mk$ - Closed set in X.

Then A is  $\hat{g}$  closed and  $\delta$ -generalized preclosed.

 $\Rightarrow$   $Cl(A) \subseteq U$  and  $Pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is Semiopen and  $\delta$ -open in X.

$$\Rightarrow Cl(A) \subseteq Pcl(A) \subseteq U$$

$$\Rightarrow$$
 Cl(A)  $\subseteq U$ 

Every  $\delta$ -open set is open set in X.

 $Int(cl(A) \subseteq int U = U$ 

$$\Rightarrow$$
 Int(Cl(A)  $\subseteq$  U

 $\Rightarrow$  Int(Cl(A)  $\subseteq$  A . since A  $\subseteq$  U

Therefore A is Semi closed in X.

The converse of the above theorem need not to be true.

### Example:3.7

Let  $X = \{a,b,c\}, \quad \tau = \{\phi,x,\{b\},\{c\},\{b,c\}\} \text{ and } \tau^c = \{\phi,x,\{a\},\{a,c\},\{a,c\}\}, \text{Semi closed} = \{\phi,x,\{a\},\{b\},\{c\},\{a,c\}\} \text{ and } \delta \text{mk-closed set} = \{\{\phi,x,\{a\},\{a,c\}\}, \{a,c\}\}, \{a,c\}\} \text{ and } \delta \text{mk-closed set} = \{\{\phi,x,\{a\},\{a,c\}\}, \{a,c\}\}, \{a,c\}\}, \{a,c\}, \{a,c\}\}, \{a,c\}, \{a,c\}, \{a,c\}, \{a,c\}, \{a,c\}, \{a,c\}\}, \{a,c\}, \{$ 

The elements  $\{b\}, \{c\}$  in Semi closed set but not in  $\delta mk$ - closed set.

### Theroem:3.6

Let  $(x,\tau)$  be a Topological Space. Prove that  $\delta mk$ -open Set is Semi open set in X.

But not conversely. (U=X)



(A High Impact Factor, Monthly, Peer Reviewed Journal)

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### Vol. 7, Issue 1, January 2018

### Example:3.8

 $X=\{a,b,c\},\$  $\tau = \{\phi, x, \{b\}, \{c\}, \{b, c\}\}$ Let and  $\tau^{c}$  $= \{\phi, x, \{a\}, \{a, c\}, \{a, b\}\},$ Semi Open={ $\phi,x, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}$ } and  $\delta mk$ -Open set={{ $\phi,x, \{b\}, \{c\}, \{b,c\}$ }.

The elements  $\{a,c\}$  in Semi open set but not in  $\delta mk$ - open set.

# Theorem:3.7

Let  $(x,\tau)$  be a Topological Space. Prove that Every  $\delta mk$ -closed set is generalized closed set. But not conversely.(U=X)

# **Proof**:

Let A be a  $\delta$ mk-closed set in X.

Then A is  $\hat{g}$  closed and  $\delta$ -generalized preclosed.

 $\Rightarrow$  Cl(A)  $\subseteq$  U and Pcl(A)  $\subseteq$  U Whenever A  $\subseteq$  U and U is semiopen and  $\delta$ -open in X.

$$\Rightarrow$$
 Cl(A)  $\subseteq_{Pcl(A)} \subseteq_{U}$ 

 $\Rightarrow$  Cl(A)  $\subseteq U$ 

Every  $\delta$ -open set is open set in X.

Then  $A \subseteq U$  and U is open in X.

Therefore  $Cl(A) \subseteq U$  Whenever  $A \subseteq U$  and U is open in X.

Hence A is generalized closed.

The converse of the above theorem need not to be true.

### Example:3.9

Let  $X = \{a, b, c\},\$  $\tau = \{\phi, x, \{b\}, \{bc\}\}$ and  $\tau^{c}$  $= \{ \phi, x, \{a\}, \{ac\} \}, \{ac\} \}$ generalized  $closed = \{\phi, x, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{bc\}\}\$  and  $\delta mk$ -closed set= $\{\phi, x, \{a\}, \{c\}, \{a, c\}\}\$ 

The element {b,c} is generalized closed δmk-closed in set but not in set Theorem:3.8

Let  $(x,\tau)$  be a Topological Space. Prove that Every  $\delta mk$ -open set is generalized open set. But not conversely.(U=X).

### Example:3.10

Let  $X = \{a, b, c\}, \quad \tau = \{\phi, x, \{a\}, \{b\}, \{a, b\}, \{bc\}\}$ and  $\tau^c = \{\phi, x, \{a\}, \{c\}, \{ac\}, \{b, c\}\},\$ generalized Open={ $\phi,x, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{bc\}\}$  and  $\delta$ mk-Open set={ $\phi,x, \{a\}, \{b\}, \{b,c\}, \{a,b\}\}$ 

The element {c} is in generalized open but in δmk-open set not set. **Properties:** 

If A and B are  $\delta$ mk-closed sets, then

1.  $A \cap B \subset A \cup B$ 



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#### Vol. 7, Issue 1, January 2018

- 2.  $(A \cup B)^c = A^c \cup B^c$
- 3.  $A^c \cap B^c = A \cap B^c$

The above properties are true for  $\delta mk$ -Open set.

#### **IV.CONCLUSION**

1. The following sets are subset of  $\delta$ mk-closed set

(i)a-closed, (ii)Regular closed, (iii)ga-closed, (iv)ag-closed

**2.** The following sets are subset of  $\delta mk$ -open set

(i)a- open, (ii)Regular open, (iii)ga- open, (iv)ag- open.

3. 8mk-closed set is subset of the following sets

(i)Semi closed, (ii)Semi preclosed, (iii)g-closed, (iv)gsp-closed, (v)pre semiclosed.

4. Smk-open set is subset of the following sets

(i)Semi open, (ii)Semi preopen, (iii)g-open, (iv)gsp-open, (v)pre semiopen.

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